3331 Assignment 4

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1. For each of the following languages, prove, without using Rice’s Theorem, whether it is in D, in SD but not in D or not in SD.  
   1. The language is in SD. We can run the dovetailing algorithm to enumerate all the strings in the language in a lexicographical order and as soon as the machine finds two strings it will halt and accept, otherwise, it will keep enumerating until it finds at least two strings.  
        
      To prove this, we reduce to, where Also, we assume that their exits a,, that decides the language generated by the reduction, , where is reduction of to.   
        
       1. Construct, where operates as follows:  
       1.1. Erase the tape  
       1.2. Write on the tape  
       1.3. Run on  
       1.4. Accept  
       2. Return  
        
      Finally, take the composite of the and, , to decide the problem.   
      If exists, decides.  
        
       - : halts on, so halts on everything. Therefore it halts on   
       at least two strings. accepts.  
        
       - : does not halt on, so halts on nothing. Therefore it   
       does not halt on at least two strings. rejects.  
        
      However, there doesn’t exist a machine that can decide, therefore, the does not exist.
   2. The language is not in SD. To prove this, we reduce to, where Also, we assume that their exits a,, that semi-decides the language generated by the reduction, , where is reduction of to.  
        
       1. Construct, where operates as follows:  
       1.1. Copy the input on a second tape  
       1.2. Erase the tape  
       1.3. Write on the tape  
       1.4. Run on for or until it naturally halts  
       1.5. If naturally halted, loop  
       1.6. Accept  
       2. Return  
        
      If exists, semi-decides.  
        
       - : does not halt on, so does not halt in steps,   
       regardless of’s size. Hence, will always reach step 1.6, which makes this   
       language infinite. accepts.  
        
       - : halts on, so halts in steps. If, will   
       reach 1.6 and halt and accept. If then the machine will naturally halt   
       at 1.4, which will lead it to enter 1.5 which will loop infinitely. Hence, this is a   
       finite set. fails to accept.  
        
      However, there doesn’t exist a machine that can semi-decide, therefore, the does not exist.
   3. The language is not in SD. To prove this, we reduce to, where Also, we assume that their exits a,, that semi-decides the language generated by the reduction, , where is reduction of to.  
        
       1. Construct, where operates as follows:  
       1.1. If then accept. Else:   
       1.2. Erase the tape  
       1.3. Write on the tape  
       1.4. Run on   
       1.5. Accept  
       2. Return  
        
      If exists, semi-decides.  
        
       - : does not halt on, so halts at 1.1. and accepts,   
       therefore, is not context-free. accepts.  
        
       - : halts on, so reaches 1.5 which implies that it   
       accepts everything. does not accept.  
        
      However, there doesn’t exist a machine that can semi-decide, therefore, the does not exist.
   4. This language is decidable, because it is empty. There does not exists a Turing machine that is distinguishable from all other Turing machines. Meaning, for every Turing machine there is at least one other Turing machine that is equivalent. Nevertheless, for every Turing machine there exists an infinite number of equivalent Turing machines.
2. For each of the languages in question 1 prove whether Rice’s Theorem can be used or not.  
   1. Rice’s theorem does not apply. The property,, does not ask about the language of the machine.
   2. Rice’s theorem does apply. The property,, of the language is the finiteness of the language. The domain of is. The property is non-trivial because it is true if the language is infinite, and false if the language is finite.
   3. Rice’s theorem does apply. The property,, of the language is whether the language falls under the context-free set of languages or not. The domain of is. The property is non-trivial because it is true if the language is not context free (determined by checking if the language belongs to), and false if the language is context-free.
   4. Rice’s theorem does not apply. The question being asked is about the machine itself not the language that the machine accepts.